

# Understanding Node Localizability of Wireless Ad-hoc Networks

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# Outline

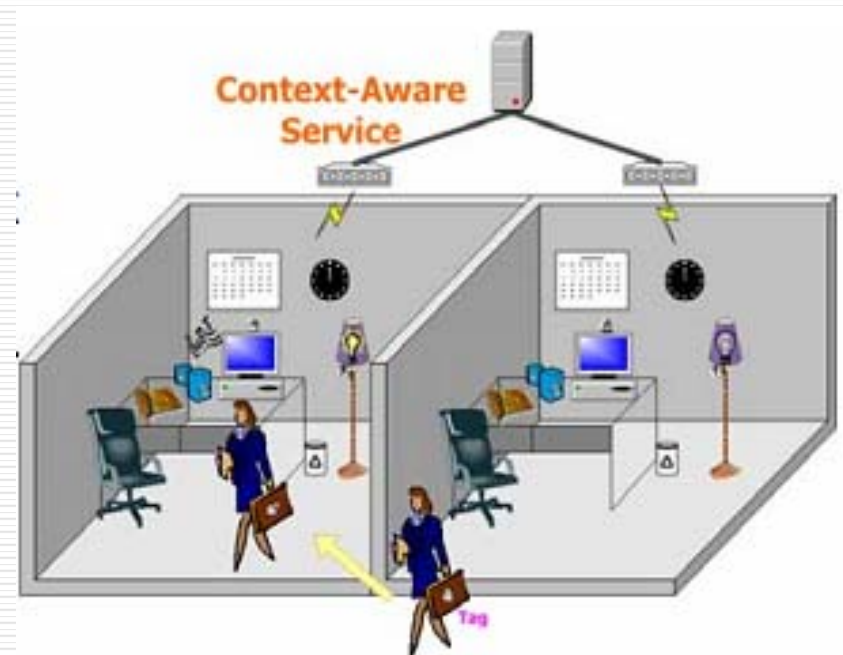
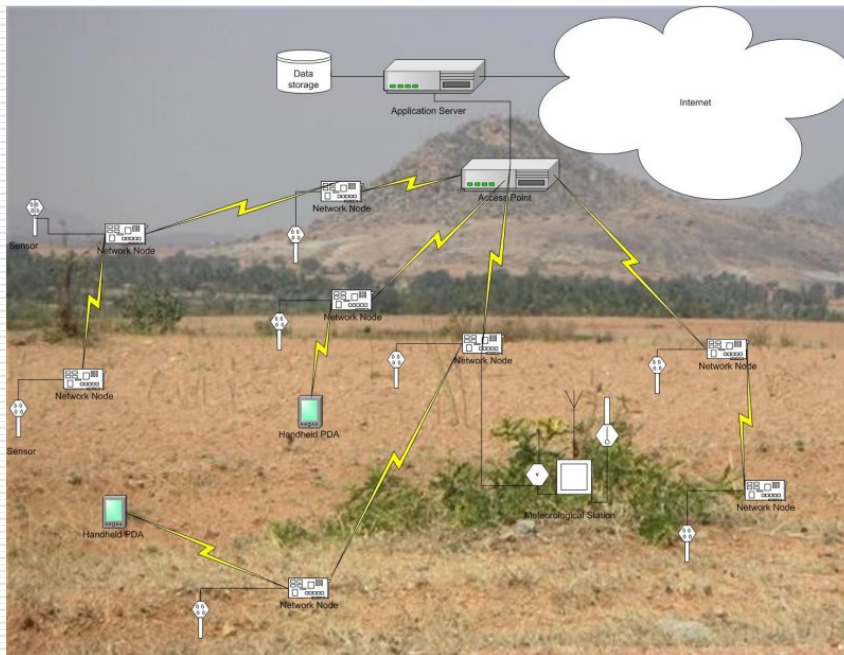
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- Introduction
  - Network Localizability
  - Node Localizability
  - Conditions for Node Localizability
  - Performance Evaluation
  - Conclusions and Future Work
-

# Localization

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- Determine the locations of wireless devices in a network



# Why locations are important?

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## Location-aware computing

### ■ Wireless sensor networks

Environmental monitoring, object tracking, ...

“*Sensing data without knowing the sensor location is meaningless.*” [IEEE Computer, Vol. 33, 2000]

### ■ Mobile computing

Mobile p2p streaming

### ■ Pervasive computing

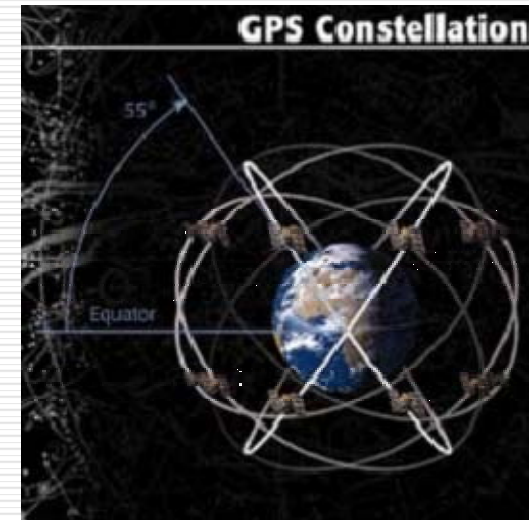
Smart space

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# Why is Localization a Non-Trivial Problem?

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- Manual configuration
  - Unscalable and sometimes impossible
- Why not use GPS?
  - Hardware requirements
  - Obstructions to GPS satellites
    - Indoor
    - Underground
  - GPS accuracy (10-20 feet) poor for short range sensors



# Network Localization

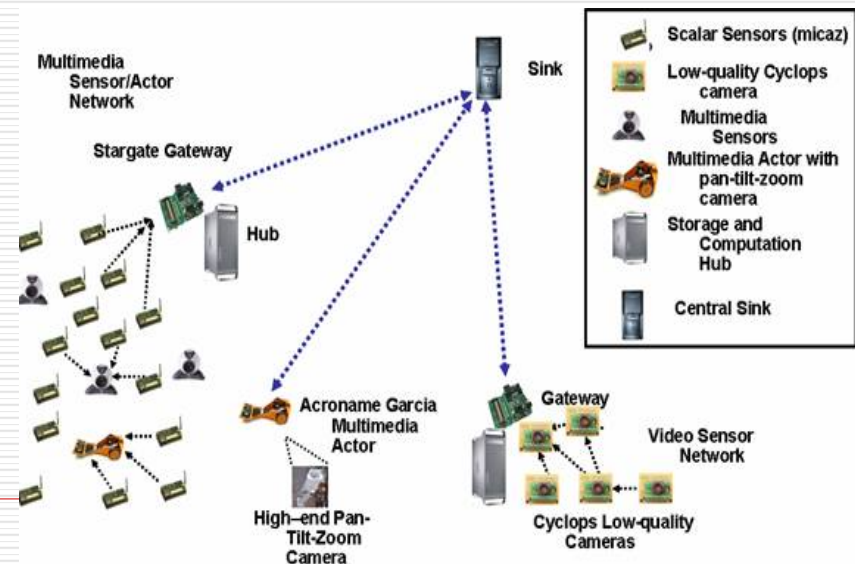
## □ Network Localization

### ■ Beacons

□ special nodes at known locations

### ■ Non-beacon nodes

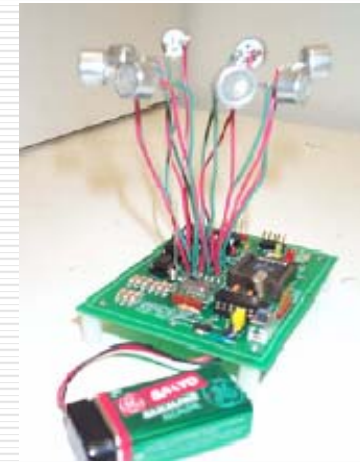
□ Distance ranging



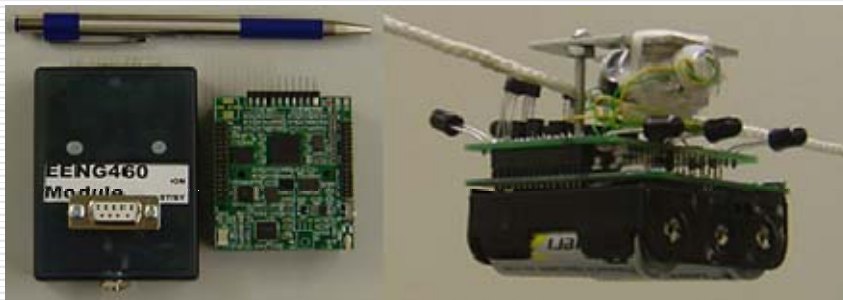
# Distance Measuring

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- Ranging techniques
  - Radio Signal Strength (RSS)
  - Time Difference of Arrival (TDoA)
- Ranging systems



UCLA medusa mote



Yale XYZ mote

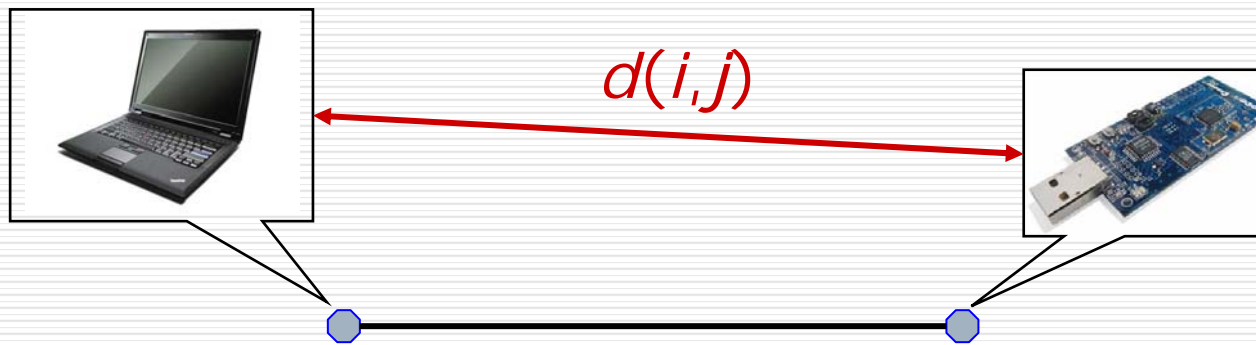


MIT Cricket mote

# Distance Graph Model

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- Distance graph  $G_N$  of a wireless network  $N$ 
  - Vertices: wireless devices (e.g., laptops, PDAs, or sensor nodes)
  - Edges: an edge connecting two vertices ( $i$  and  $j$ ) if the distance  $d(i,j)$  between corresponding nodes can be measured



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# Network Realization

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## Definition

A *realization* of a graph  $G$  is a function  $p$  that maps the vertices of  $G$  to points in a Euclidean space.

Generally, realizations are referred to the *feasible* ones that respect the pairwise distance constraints

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# Localizability

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## Definition

A network is *localizable*  
if it has a **unique** realization in some  
metric space.

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# Localizability

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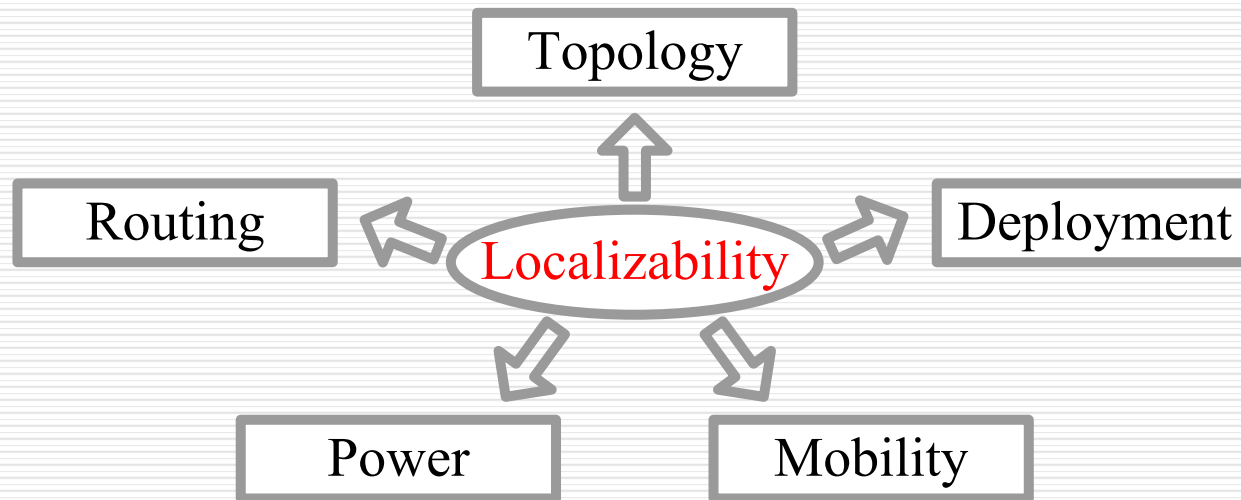
## □ **Localizability** V.S **Localization**

- If a network is **NOT** localizable, by no means it can be localized.
  - If a network is localizable, it can be localized in theory (but may be computationally infeasible).
-

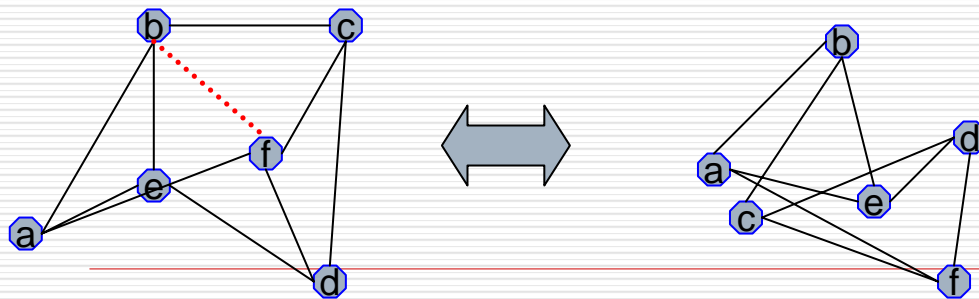
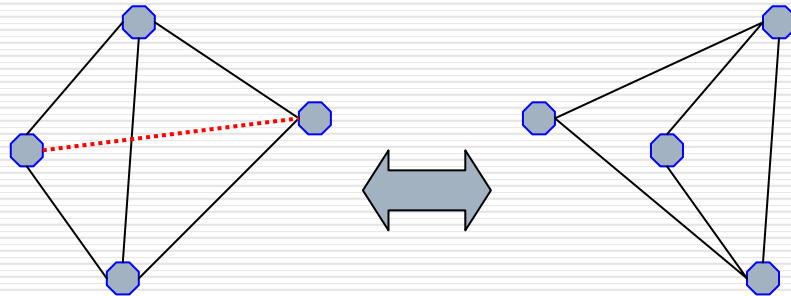
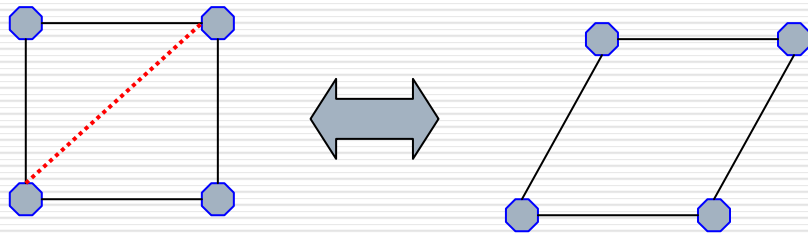
# Why Localizability is Important?

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- Being aware of localizability helps:



# Localizability and Graph Rigidity



## Solution:

G must be *rigid*.

G must be 3-connected.

G must be *redundantly rigid*:  
It must remain rigid upon  
removal of any single edge.

# Localizability and Graph Rigidity

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Global rigidity (by Jackson and Jordan, 2003)

A graph is generically globally rigid in 2D plane iff. it is 3-connected and redundantly rigid.

The necessary and sufficient condition for localizability.

Network localizability (Eren, 2004)

A network is uniquely localizable iff. its distance graph is globally rigid and it contains at least three beacons.

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# Localizability Test Algorithm

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- Network localizability can be tested
    - Polynomial time to the graph size
      - Rigidity:  $O(n^2)$  by the pebble game algorithm by Jacobs and Hendrickson (1997)
      - Redundant rigidity:  $O(n^2)$  algorithm by Hendrickson (1991)
      - 3-connectivity:  $O(n)$  algorithm by Tarjan (1972)
  - So far, it seems ...
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# Node Localizability

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## Observations

Almost **all the time** the network is **NOT** entirely localizable.

A large portion, on average nearly **80%**, of nodes are actually localizable.



# Node Localizability

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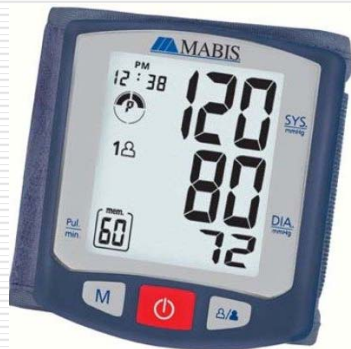
## □ Node localizability

- To answer the question that *whether a particular node has a unique location.*
  - Single node V.S entire network
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# Why Node Localizability is Important?

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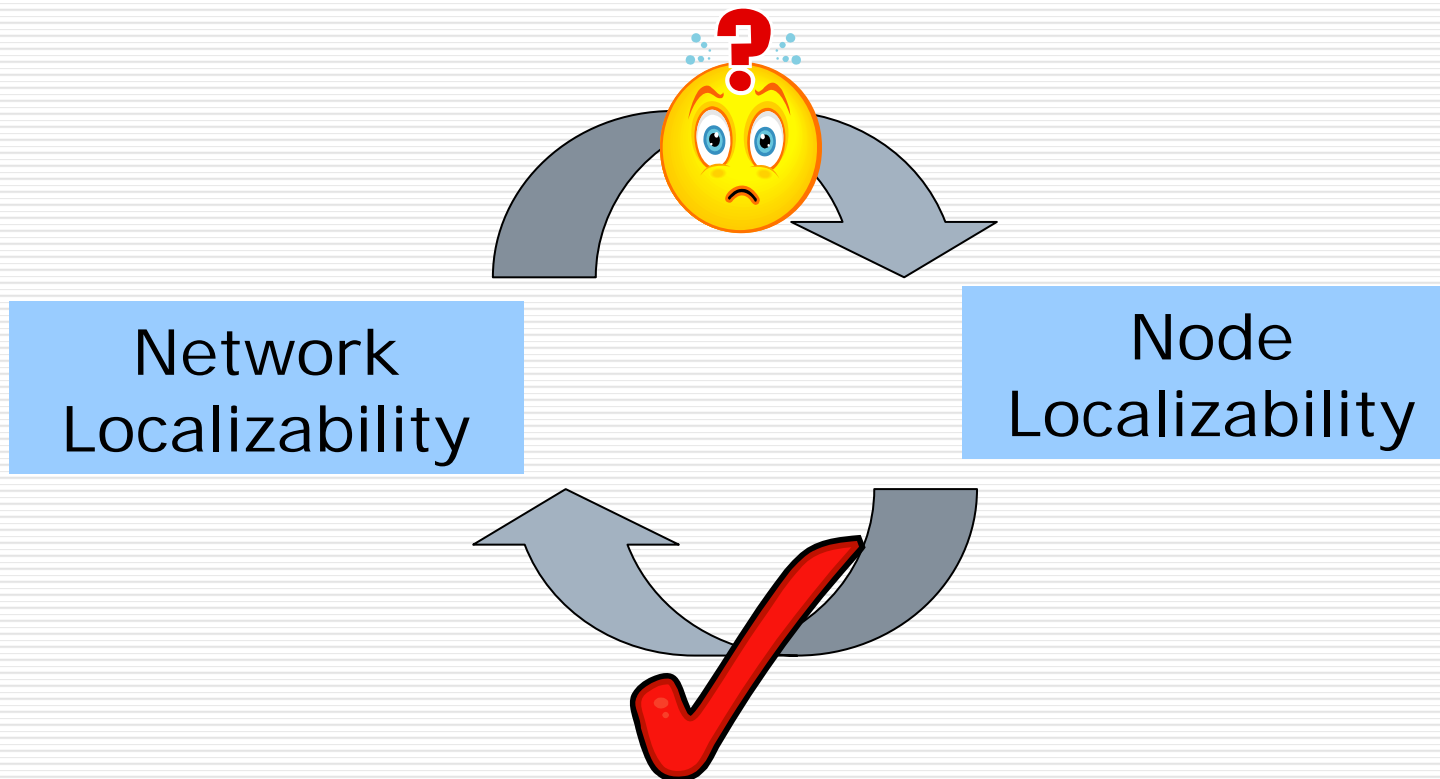
- Partially localizable networks
  - They are not localizable.
  - A portion of nodes have unique locations while others do not.
- Application
  - A portion of nodes draw remarkable attentions



# Node Localizability

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- Which one is harder?



# Why Node Localizability Is Difficult?

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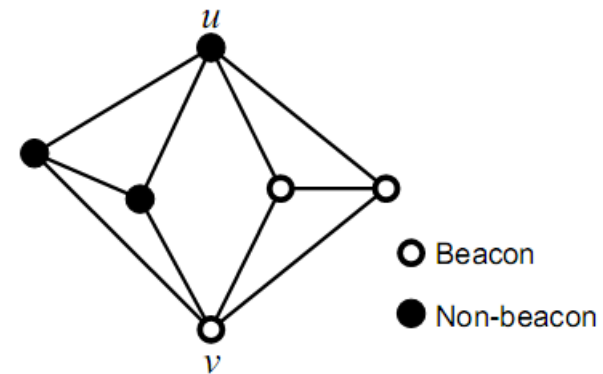
- A straight-forward solution
    - Find a sub-network that is localizable
    - Identify all nodes in the sub-network localizable
  
  - Correct? **YES, BUT...**
-

# Why Node Localizability Is Difficult?

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## □ Missing localizable nodes

- $G$  is not 3-connected
- $u$  is localizable



Some conditions *essential* to network localizability are *no longer necessary* for node localizability.

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# Conditions for Node Localizability

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## □ Necessity

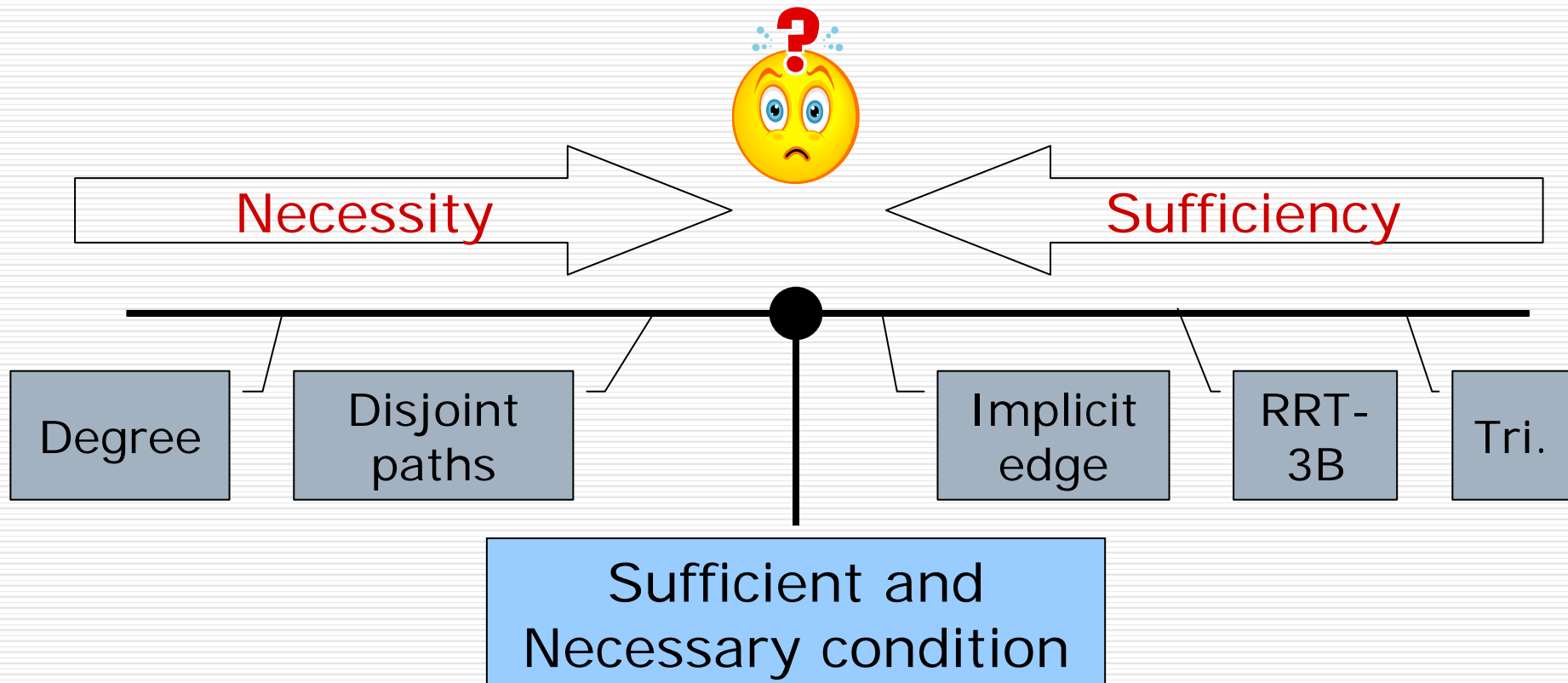
- Degree = 3
- 3 vertex-disjoint paths to 3 distinct beacons [Goldenberg, 2005]

## □ Sufficiency

- Trilateration
  - Localizable sub-network [Goldenberg, 2005] .
  - Implicit edge [Eren, 2005]
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# Previous work

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# Necessary Conditions

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- 3 vertex-disjoint paths (3P)

- Goldenberg, 2005

- Redundant Rigidity (RR)

- *In this study, 2009*

- If a vertex is localizable, it is included in the redundantly rigid component of beacon nodes.

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# Necessary Conditions

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## □ Necessity

- The combination of 2 necessary conditions is also a necessary condition

RR-3P

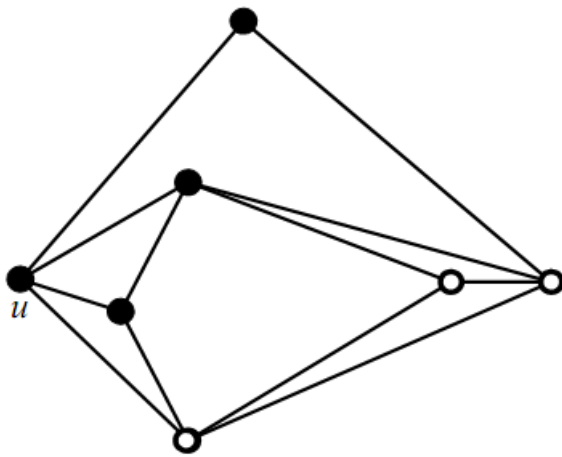
### Theorem

In a distance graph  $G = (V, E)$  with a set  $B \subset V$  of  $k \geq 3$  vertices at known locations, if a vertex is localizable, it is included in the **redundantly rigid** component that contains  $B$  and has **3 vertex-disjoint paths** to 3 distinct vertices in  $B$ .

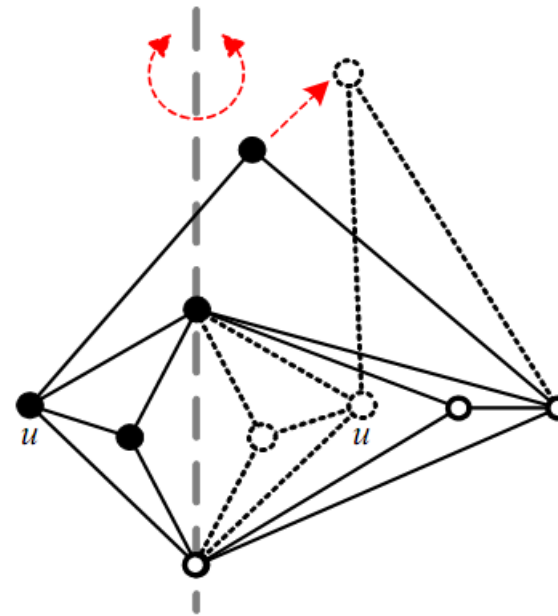
# Necessary Conditions

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□ RR-3P is **NOT** sufficient



(a) The vertex  $u$  satisfies RR-3P.



(b)  $u$  suffers a discontinuous flexing.

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# Sufficient Conditions

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## □ RR3P condition

### Theorem

In a distance graph  $G = (V, E)$  with a set  $B \subset V$  of  $k \geq 3$  vertices at known locations, a vertex is localizable if it belongs to the redundantly rigid component of  $B$  in which it has 3 vertex-disjoint paths to 3 distinct vertices in  $B$ .

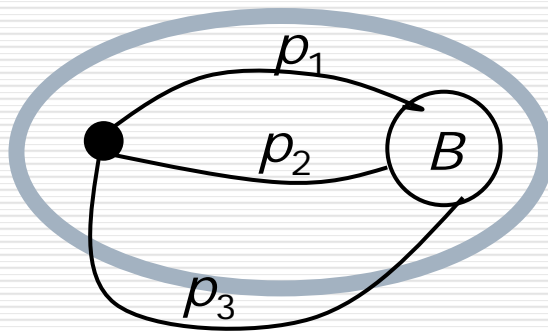
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# Summary (1)

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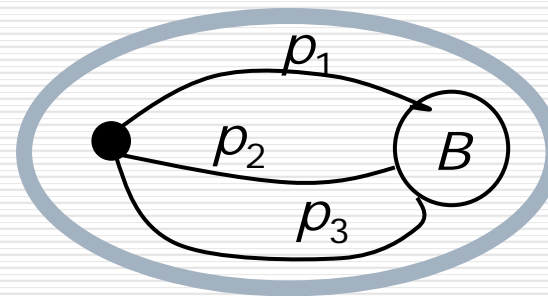
Necessity

RR-3P



Sufficiency

RR3P

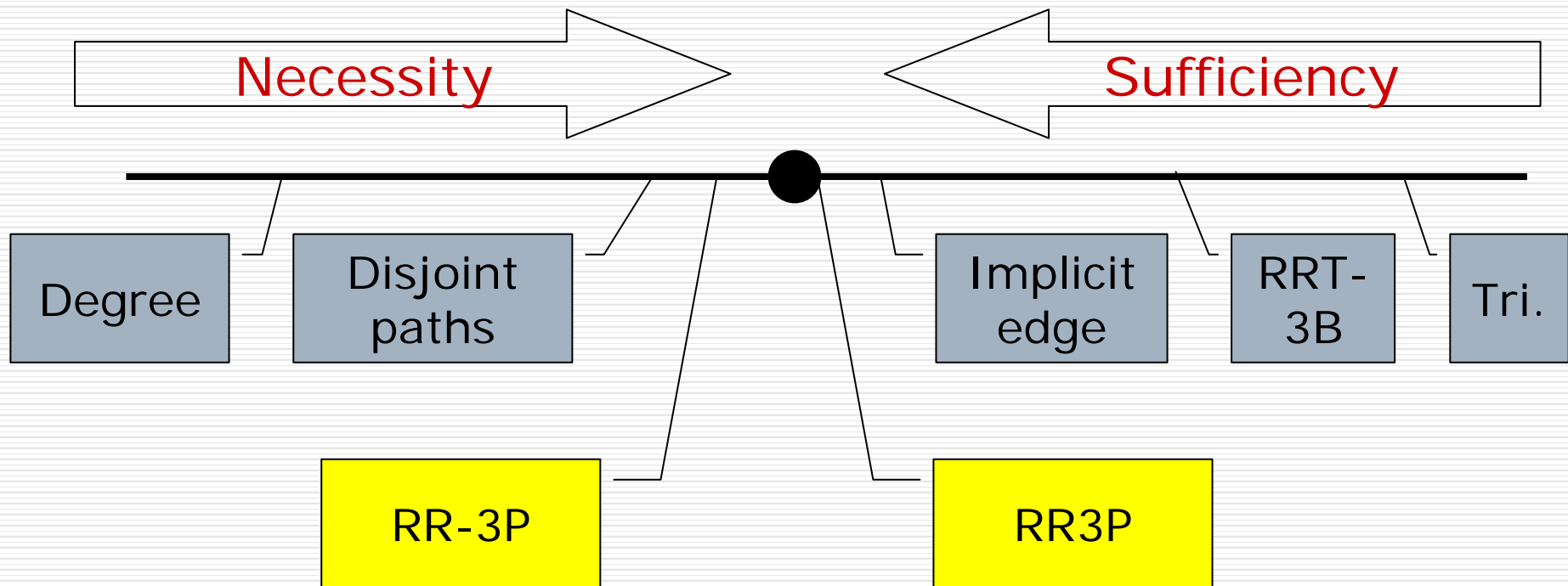


All paths are strictly included

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# Summary (2)

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# Outline

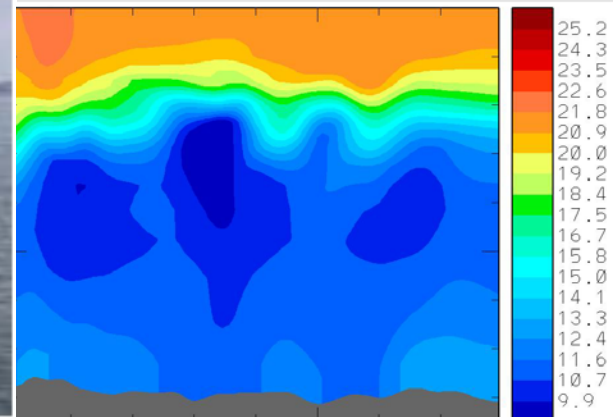
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# System Description

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- Sea monitoring WSN
  - 100 wireless sensors
  - Environmental Data
    - temperature, humidity, illumination...
  - Localization: Trilateration



# Observation (1)

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## □ Observations

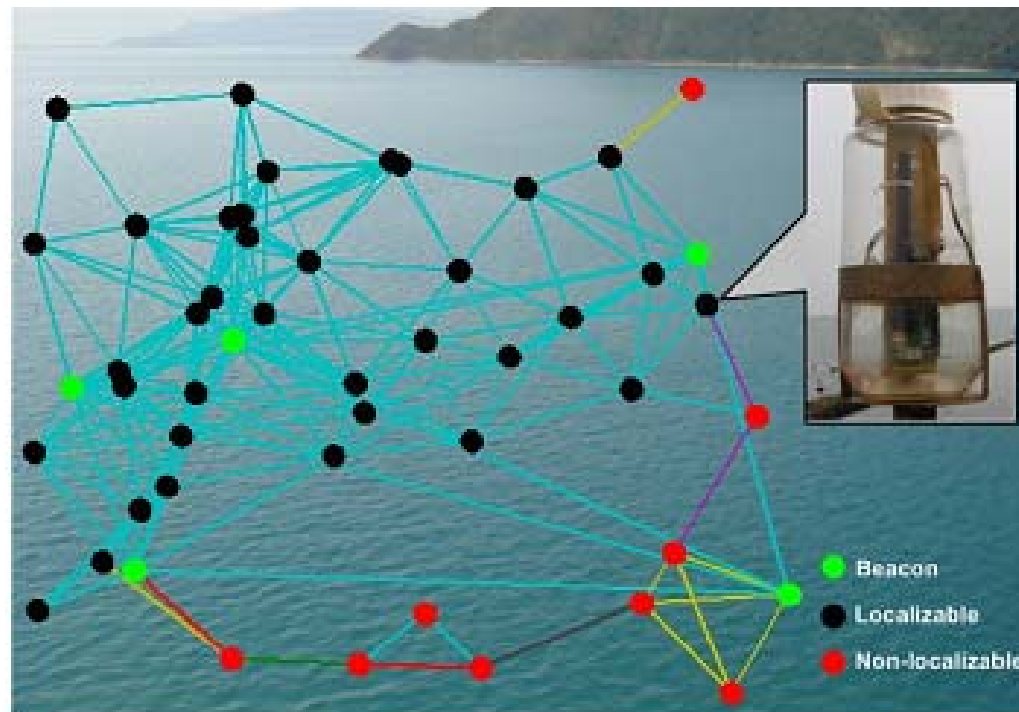
- Almost all the time the network is **NOT** entirely localizable.
- A large portion, on average nearly **80%**, of nodes are actually localizable
- Specifically, **90%** of network topologies have at least **60%** of nodes localizable

## □ The importance of node localizability.

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# Observation (2)

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# Simulations

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## □ Metrics

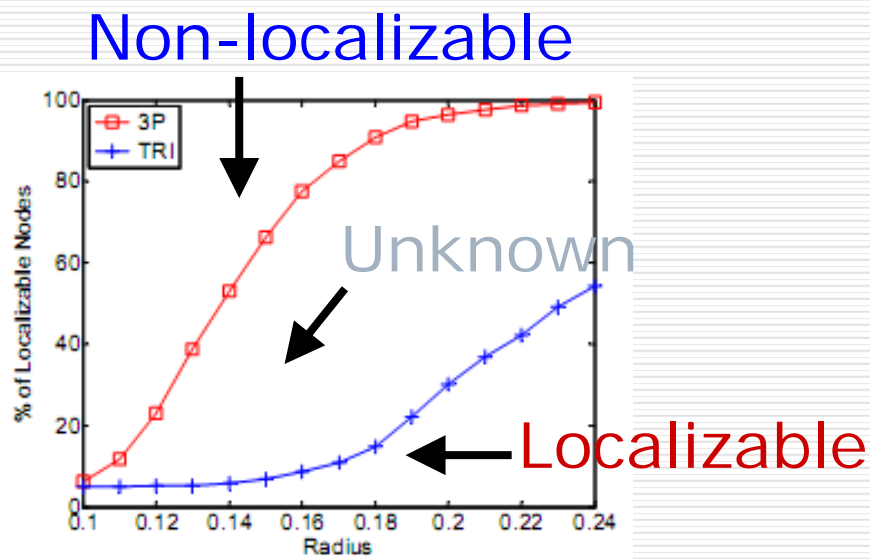
- the number of nodes that can be identified

## □ Comparison

- Necessary conditions
    - **3P** V.S. **RR-3P**
  - Sufficient conditions
    - **TRI** V.S. **RR3P**
-

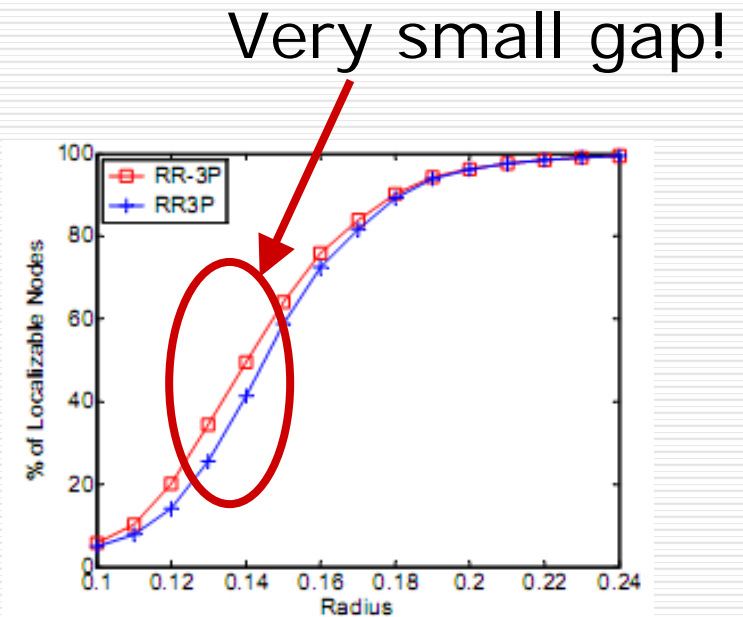
# Results(1)

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(a) The capability of 3P and TRI.

3P and TRI



(b) The capability of RR-3P and RR3P.

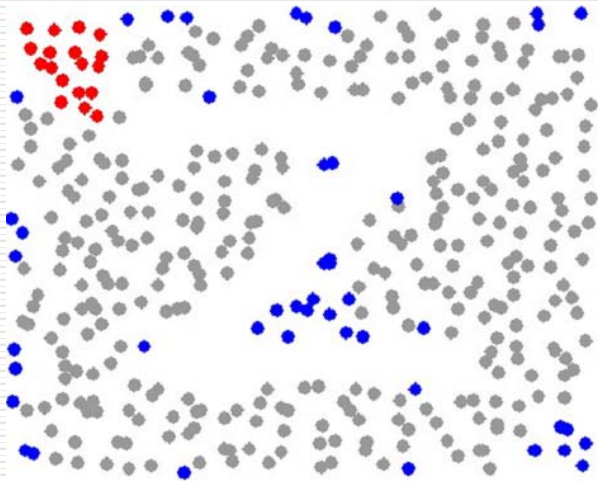
RR-3P and RR3P

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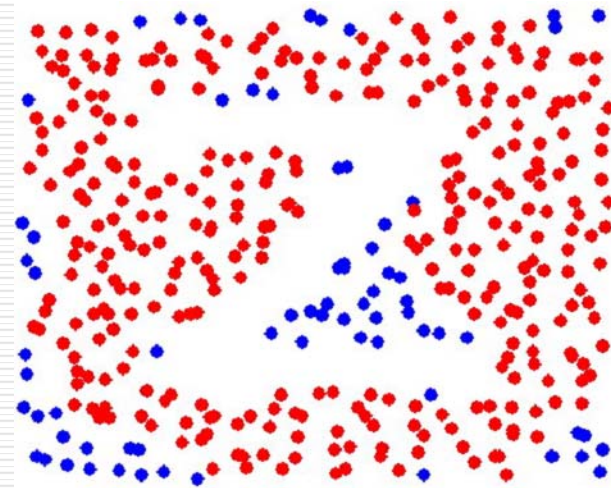
## Results(2)

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- network with a “Z” hole
  - **Blue**: non-localizable
  - **Red**: localizable
  - **Grey**: unknown



3P and TRI



RR-3P and RR3P

# Results(3)

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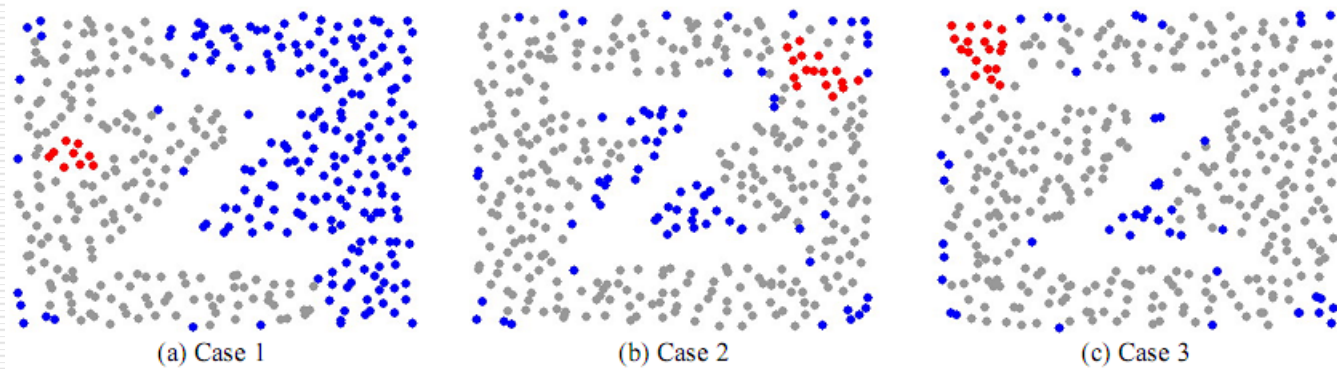


Figure 15: Testing 3P and TRI on network instances with "Z" holes.

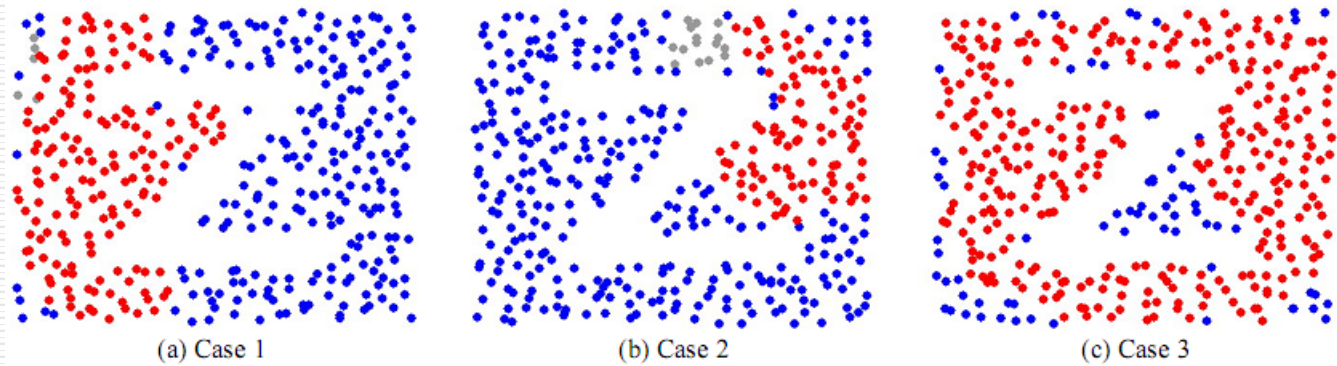


Figure 16: Testing RR-3P and RR3P on network instances with "Z" holes.

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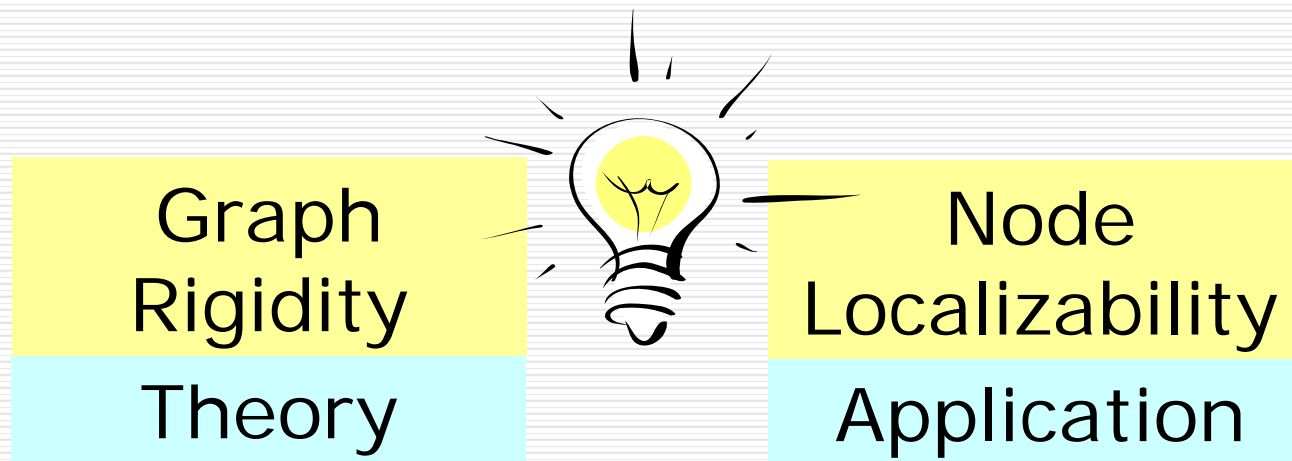
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# Conclusion

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- ❑ Limitations of network localizability
- ❑ Node localizability
- ❑ Necessary and sufficient conditions



# Future Work

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- Localizability under noisy ranging
  - Localizable ???
    - perfect ranging
    - noisy ranging
  - Robustness of localizability testing





**Thanks.**  
**Any questions?**

